HISTORY OF EQUALIZATION 1860–1980

DAVID FALCONER, CARLETON UNIVERSITY (ddf@sce.carleton.ca)

INTRODUCTION

Operators of the first transatlantic telegraph cable in 1858, and subsequent cables from the mid-1860s, noticed that they had to transmit Morse code dots and dashes very slowly to be understood at the other end.[1] This phenomenon, due to smearing out of pulses by the capacitive effects of very long cables, had been predicted by William Thomson (more about him below), and was one of the first instances of the need for equalization of digital signals.

Digital communication by modulated serially transmitted pulses generally requires equalization if the communications medium distorts and spreads the pulses so that they interfere with one another, causing intersymbol interference (ISI). Such a channel is called a dispersive, or frequency-selective, channel; its attenuation and delay characteristics vary over the band of frequencies spanned by the transmitted signal. Equalization is the remedial signal processing, employed at the receiver and/or the transmitter, to suppress the ISI impairment.

A search for "equalization" in IEEE Xplore results in over 11,000 publications. In this article, I attempt to give a historical perspective on the main research and development threads up to 1980 — arguably the "golden age" of equalizer research, with some mention of their influence on developments since then. Some of the material is colored by my personal involvement in equalization research. In portraying the history of a field with so many facets and contributors, the hardest task is deciding what to leave out. I apologize to those whose ideas and publications I have omitted due to lack of space. Detailed technical overviews of equalization developments in this period are found in the tutorial and overview papers by Proakis and Miller [1] and Qureshi [2], and in the textbooks of Lucky, Salz, and Weldon [3], Proakis [4], and Gitlin, Hayes and Weinstein [5]. For reasons of brevity, most post-1980 advances in equalizer theory and algorithms have been omitted, with the hope that these may be covered in a later history paper.

EARLY HISTORY OF EQUALIZATION: 1860–1940

Probably the earliest need for equalization for electrical signals was in the 1860s for submarine telegraph cables. William Thomson, an Irish-born physicist and engineer (later Lord Kelvin) mathematically modeled the electrical attenuation and dispersion inherent in electrical transmission through long cables, concluding that distributed capacitance in the cables was responsible. Thomson recommended appropriate cable composition and dimensions, and invented, among other devices, a novel "automatic curb sender" electromechanical transmitting device to minimize the effects of cable dispersion. This device transmitted each original pulse as two alternate polarity current pulses, thus shortening the effect of the cable's response — a form of line coding, related to what today we call Manchester coding. Some submarine cable systems also used signal shaping — an early form of linear transmitter equalization.

The next major contributor to equalization theory and practice was the English electrical engineer, mathematician and physicist Oliver Heaviside. He formulated and used Maxwell's equations to develop transmission line theory, and from it, in 1887, deduced that adding inductors at regular intervals along a transmission cable would reduce its attenuation and dispersion. This amounted to a form of tuning that linearly equalized the cable.

Heaviside's theory of distributed inductive equalization of cables was followed up and patented by George Campbell of AT&T and Michael Pupin of Columbia University in 1900. Deployed by the Bell System and other telephone companies starting in the early 20th century, this technique was called *inductive loading*. It made possible long distance telephony and relatively long telephone subscriber loops. In the 1920s Campbell and others at Bell Labs and elsewhere developed concepts for the synthesis of linear filters from inductors, capacitors and resistors. In particular O. J. Zobel in 1928 and H. W. Bode in 1938 showed in *Bell System Technical Journal* papers how such linear lumped-element filters could be designed and adjusted to equalize for the linear amplitude and phase distortion in telephone circuits.

EQUALIZER OPTIMIZATION CRITERIA

Our present understanding of the ISI problem addressed by equalizers starts with the mathematical representation of a baseband pulse amplitude modulated (PAM) data signal consisting of a serial stream of transmitted or received data symbols,

$$s(t) = \sum_n d_n h(t - nT), \tag{1}$$

where $\{d_n\}$ represent data symbols from a finite alphabet, T is the time interval between adjacent symbols, so $1/T$ is the symbol rate, and $h(t)$ represents a transmitted or received pulse waveform. If $s(t)$ is observed at a receiver, $d_n h(t - nT)$ represents the response of a data symbol d_n transmitted at time nT to the combination of transmitter filter, channel, and receiver filter. If $s(t)$ is sampled at $t = kT$ to try to recover the kth data symbol d_k, the ISI is represented by the sum

$$ISI(k) = \sum_{n \neq k} d_n h(k - n)T). \tag{2}$$

A linear equalizer may be part of the receiver filter, the transmitter filter, or both. These equations may also represent a baseband equivalent signal before linear modulation or after demodulation; in this case $\{d_n\}$ and h(t) may be complex-valued, representing in-phase and quadrature-phase signal components.

Harry Nyquist [6, 7] pointed out the (in hindsight, obvious) criterion for the ISI to be zero when $s(t)$ is sampled at the symbol rate $1/T$. The criterion, known as the Nyquist criterion for zero ISI, expressed in the time domain, is that the overall impulse response $h(t)$ satisfies

[1] *The first message, from Queen Victoria to President James Buchanan, took over 16 hours to transmit. The 1858 cable worked for only a few weeks before failing completely. The September 2008 history paper in this magazine by Jerry Hayes, the 2001 book* History of Telegraphy *by K. Beauchamp, and ComSoc's 2002* A Brief History of Communications *detail the emerging telecommunications technologies, including means of equalization, in early cable systems.*

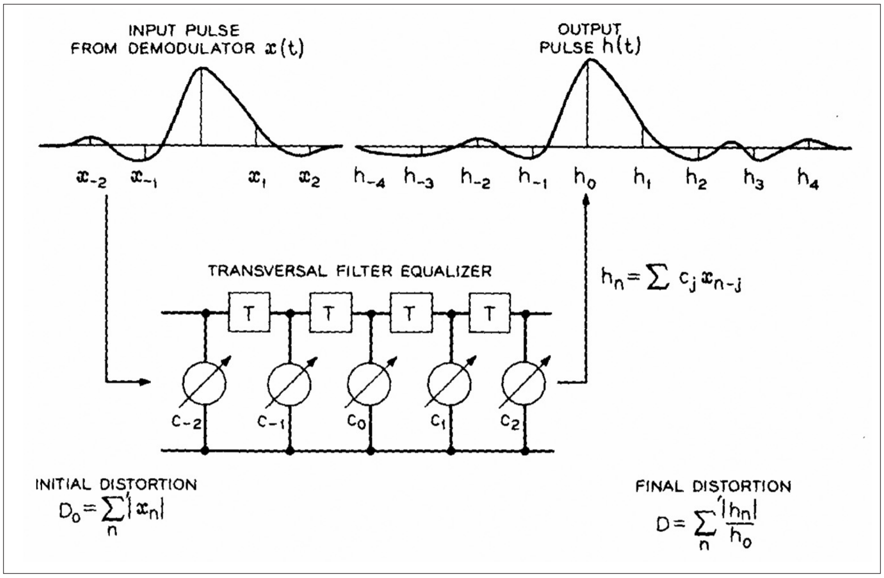

Figure 1. *Transversal filter equalizer, with an example of a pulse shaped to satisfy the Nyquist criterion for zero ISI. The primes on the summations mean that the terms with zero index are omitted. (Reproduced from [10]).*

$$h(nT) = 0 \text{ for } n \neq 0. \tag{3a}$$

The equivalent statement in the frequency domain is

$$\hat{H}(f) = \sum_{n=-\infty}^{\infty} H\left(f - \frac{n}{T}\right) = \text{constant for any frequency } f, \tag{3b}$$

where $H(f)$, the Fourier transform of $h(t)$, is the overall frequency response. From this criterion Nyquist concluded that ISI-free baseband data transmission at symbol rates above twice the channel bandwidth is impossible.[2]

The simplest conceptual version of equalization is known as zero-forcing, where the filtering satisfies Eq. 3. At frequencies for which the channel response dips toward zero, the noise spectrum is amplified by the equalizer. This so-called *noise enhancement problem* can lead to high error rates on highly frequency-selective channels for detectors which have been optimized for white noise. Minimization of the mean squared error (MSE), consisting of ISI and noise at the equalizer's output, is an alternative to the zero-forcing criterion, and partially addresses the noise enhancement problem.

Donald Tufts, of Harvard University, provided the analytical framework for optimum zero-forcing and minimum MSE equalization for a channel with additive white noise and a given frequency response [8]. That paper and others also considered splitting linear equalization between transmitter and receiver, and found expressions for the jointly optimum frequency responses. These theoretical results showed that an optimum linear equalizer can be realized as cascade of a filter matched to the received pulse, followed by a symbol rate sampler and a transversal filter, with taps spaced at symbol intervals. The sampled output of such a transversal equalizer with N taps, with input samples $\{y(nT)\}$ and coefficients $\{c_n\}$, can be represented as

$$z(kT) = \sum_{n=0}^{N-1} c_n y((n-k)T. \tag{4}$$

Typically, the number of tap coefficients should be on the same order as, or exceed, the number of sampling intervals spanned by the overall channel impulse response.

While the 1960s theoretical papers were yielding useful insights into equalizers' optimized capabilities and performance, data rates for voiceband telephone services and certain military digital radio systems were being pushed toward channel bandwidth limits, to the point where ISI was starting to limit performance. Practical equalizer realizations were necessary, as well as the ability to adapt equalizers to channel responses.

AUTOMATIC AND ADAPTIVE EQUALIZATION

For precise equalization of initially unknown channel responses, the tap coefficients of transversal filter equalizers must be adjusted to their optimum values by means of an initial training procedure and/or during actual data transmission. Optimum tap coefficients can be computed from a channel response that is estimated during training, or equalizer coefficients can be obtained directly in an iterative fashion, using known training symbols as references. For early adaptive equalizers the iterative approach was favored, whereby each tap coefficient, c_n was then incremented by an amount depending on the measured response:

$$c_n^{(k)} = c_n^{(k-1)} + \text{increment at the } k\text{th iteration for the } n\text{th tap.} \tag{5}$$

In 1963 Bob Lucky at Bell Labs devised an iterative "steepest descent" technique to adapt or "train" the tap coefficients of a transversal filter equalizer to minimize the peak ISI for any given channel before the start of actual data transmission. At the kth iteration Lucky's "automatic equalizer" [9] used the response of the combined channel and equalizer, measured by the response to a transmitted training pulse. The increment to any tap coefficient at each iteration was proportional to the opposite sign of the corresponding estimated response sample. This simple iterative algorithm is equivalent to a gradient descent algorithm for minimizing the peak value of ISI. Within the limitations of finite transversal equalizer length, it is an implementation of the zero-forcing criterion. Figure 1, from Lucky's 1966 paper [10], shows the structure of a transversal equalizer, and illustrates the equalizer's effect on a pulse received from a channel.

Lucky and his colleagues at Bell Labs implemented this automatic equalizer using digital logic and hundreds of relays to control 13 adjustable tap coefficients implemented as ladder attenuation networks. Bob Lucky describes in *A Brief History of Communications* how one could follow the progress of the equalizer's self-adjustment by the sound

[2] *It has since been realized that some practical transmission media, such as unfiltered cables or wire pairs, do not have well defined bandwidths, so symbol rate upper limits are correspondingly ill defined.*

of the relays clicking frantically until the minimum was reached, at which point only sporadic clicks were heard. I had a summer job at Bell Labs in 1964, and remember seeing this prototype automatic equalizer clicking away in a refrigerator-sized cabinet.

Seeking ways to adjust the equalizer during actual data transmission, Lucky devised an "adaptive equalizer" iterative algorithm, which used the estimated data symbols $\{\hat{d}_n\}$ and differences between them and the equalizer's outputs to determine the required tap coefficient decrements [10].

These iterative algorithms attempted to converge to a set of equalizer tap coefficients that minimized the peak ISI, without taking noise into account; they were in effect zero-forcing equalizers. Most present-day adaptive equalizers are based on the MSE criterion: minimizing the expectation or the average over time of the square of the error $(z(nT) - d_n)$ at the equalizer's output, which includes noise. This provides a means of limiting the noise enhancement problem to some extent. One of the simplest and most popular algorithms for doing this is the *least mean square (LMS) algorithm*, which was introduced in 1960 by Bernard Widrow and Marcian Hoff at Stanford University in a paper on adaptive switching circuits [11]. David Coll at the Defence Research Telecommunications Establishment and Donald George at Carleton University in 1965 [12] proposed maximization of the equalizer's output signal-to-noise ratio, where "noise" includes residual ISI. They, and also Lucky and his colleague Harry Rudin, applied the LMS algorithm in implementing this adaptive equalizer in hardware. The increment to the set of equalizer tap coefficients in the LMS iteration is proportional to the negative gradient of the squared error at time n, expressed as $-(z(nT) - \hat{d}_n)\, x((n-k)T)$ where $\{\hat{d}_n\}$ are pseudorandom training symbols during start-up, or receiver decisions during data transmission, and $\{x(nT)\}$ and $\{z(nT)\}$ are equalizer input and output samples, respectively.

1960S AND 1970S: DEVELOPMENT OF EQUALIZATION TECHNIQUES FOR VOICEBAND TELEPHONE CHANNELS

These equalizer adaptation developments came at a crucial period for the nascent data communications industry. At the time, the primary commercial telecommunications system was the voiceband telephone network. Information technology applications requiring data sharing among remote locations were rapidly expanding. The 1968 Carterfone decision by the U.S. Federal Communications Commission opened the door toward unrestricted access to this network with customer-owned equipment. There was increasing demand for higher data rates and improved reliability.

Between a pair of customer terminals or modems, a voiceband telephone channel appears as a nominal passband filter spanning frequencies from a few hundred Hertz to a little over 3 kHz. Amplification used in telephone network facilities generally ensures a high signal-to-noise ratio at the receiving end — typically on the order of 20 dB or more. The signal may pass through several stages of channel banks with filtering, analog or digital pulse code modulation (PCM) transmission, multiplexing, and demodulation in each stage. Until the 1980s, the filters were mainly analog, and varied considerably due to aging, temperature variations, etc., leading to unpredictability in the channel frequency response. Other impairments such as nonlinearities, phase jitter, frequency offset and impulse noise were also present, along with the usual receiver thermal noise. The bandwidth and noise characteristics of long distance voiceband telephone channels limit voiceband modems to no more than several tens of kilobits per second data rates. Their relatively sophisticated and necessary modulation, coding, adaptive equalization, synchronization, and filtering operations fitted well with the emerging digital signal processing capabilities of the 1960–1980 period. It is not surprising that many advances in communication theory and communication signal processing were spurred by the expanding and competitive voiceband modem arena at that time.

I had the privilege of joining the Data Theory group at Bell Labs, under Bob Lucky and Jack Salz, at the end of 1967; many theoretical and practical equalization developments originated from this group, which worked closely with modem devel-

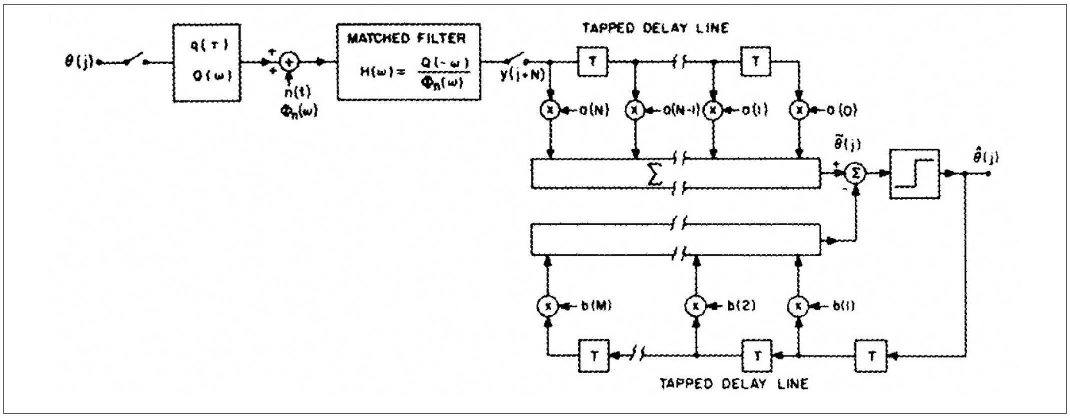

Figure 2. *Decision feedback equalizer. (Reproduced from [16].)*

opers as well as with other researchers at Bell Labs. At the same time, other notable contributors to equalizer and modem development were at companies like Codex Corporation, IBM, Intertel, and NEC, as well as at many universities.

Voiceband modems with data rates of 4.8 kb/s and 9.6 kb/s, equipped with adaptive equalizers, started to appear in the late 1960s. Early high-speed modems tended to use vestigial sideband modulation (VSB) or single sideband modulation (SSB), and baseband adaptive equalizers of one of the types just described. In the 1970s high-speed modems based on quadrature amplitude modulation (QAM) or combined amplitude and phase modulation (AM-PM) prevailed due to their greater robustness to phase, frequency, and synchronization impairments.[3] Such signals can be represented conveniently by complex-valued waveforms, and the corresponding equalizer tap coefficients can also be considered complex-valued. The same expressions (now with complex variables) for the Nyquist criterion and equalizers apply, and $H(f)$, which appears in the frequency domain Nyquist criterion, is now a bandpass frequency response, centered at some frequency other than zero.

Rich Gitlin, Ed Ho, and Jim Mazo at Bell Labs pointed out that linear equalization could be done on bandpass modulated signals, introducing the concept of an adaptive "passband" equalizer for AM-PM or QAM signals [13]. Following work on decision-directed phase estimation and adaptive equalization by Hisashi Kobayashi [14], a March 1976 *BSTJ* paper by me, on the combination of decision-directed adaptive bandpass equalization and carrier recovery, showed an advantage of carrier recovery for demodulation *after* passband equalization. This approach has been followed in many high-speed voiceband modems to minimize delay in estimating and removing the effects of phase jitter.

If the equalizer's input sampling rate is greater than the symbol rate (and the tap delays are correspondingly less than the symbol interval), the equalizer is termed *fractionally spaced*, and has better control over the overall frequency response. Fractional spacing was proposed by Bob Lucky in a 1969 Allerton Conference paper. The performance benefits, including reduced sensitivity to timing phase error and more efficient use of a fixed number of tap coefficients, were quantified independently in papers by Loïc Guidoux of Télécommunications Radioélectriques et Téléphoniques (TRT) and Gottfried Ungerboeck of IBM, and a patent by Shahid Qureshi and G. David Forney Jr. of Codex. Codex claimed the first commercial fractionally spaced equalizer. Subsequently, fractionally spaced equalizers became widely used.

NONLINEAR EQUALIZATION TECHNIQUES FOR HIGHLY DISPERSIVE CHANNELS

For digital transmission on channels with deep frequency response "valleys," linear equalizers yield poor performance due to the noise enhancement problem. Radio channels and long cables and twisted copper pairs are examples of such channels. Alternatives to linear equalization then become of interest in dealing with the ISI problem on such channels. Decision feedback equalization (DFE) is such an alternative. A landmark paper on the theoretical capabilities of DFE by Robert Price of Sperry Rand Research Center in 1972 recounted the long history of DFE up to that point [15], starting with a 1919 patent assigned to Robert Mathes of Western Electric. A DFE has an additional feedback transversal filter that subtracts the effects of previous received symbol decisions. In this way, ISI from already detected symbols is eliminated. Adaptation of the forward and feedback filters of DFEs follow the same pattern as for linear equalizers. The structure of a DFE, from [16], is shown in Fig. 2.

DFEs largely avoid the noise enhancement problem, but present another potential problem associated with feeding back filtered incorrect detector decisions; errors caused by noise may then propagate, causing further errors. Thus, DFEs are viewed with some caution by many system designers. One approach to avoid the error propagation problem is to put the symbol feedback at the *transmitter*, where errors are impossible. This approach was proposed by Tomlinson [17] and by Harashima and Miyakawa [18]. The so-called Tomlinson-Harashima precoding uses the feedback filter tap coefficients in an inverse filter configuration at the transmitter, together with a modulo-N adder, where N is such that the range of data symbol amplitudes is within $\pm N/2$. In papers published in the 1990s, Vedat Eyuboglu and Dave Forney of Codex showed that if channel knowledge is available at the transmitter, Tomlinson-Harashima pre-

[3] *QAM here refers to amplitude modulation of in-phase and quadrature carriers independently, thus creating a two-dimensional signal constellation on a rectangular grid. AM-PM creates a two-dimensional constellation without restriction to a rectangular grid. Design of the signal constellation depends on factors such as robustness to noise and phase jitter.*

coding combined with trellis coding is close to an optimum equalization/coding method for dispersive channels. Precoding approaches are used in high-speed voiceband modem standards such as International Consultative Committee for Telephone and Telegraph (CCITT) V.34 and also in 10 Gb/s 10GBASE-T Ethernet twisted pair transmission.

In 1971–1972 G. Dave Forney derived a receiver that outputs the sequence of transmitted data symbols which, with *maximum likelihood*, were received together with additive white Gaussian noise from a dispersive linear channel. Such a receiver offers close to the minimum error probability. His paper [19] showed that such a maximum likelihood sequence estimation (MLSE) consists of a so-called whitened matched filter (WMF),[4] followed by a recursive nonlinear processor called a Viterbi algorithm. The Viterbi algorithm, which in this case operated on the output of the WMF and made use of knowledge of the overall response, had been invented by Andrew Viterbi in the mid-1960s as a maximum likelihood decoder for convolutional codes. As recounted in his comments in this magazine to a March 2009 history paper on partial response by Kobayashi, Forney had started to think about applying the Viterbi algorithm to partial response signals[5] in the mid-1960s, which led eventually to his 1972 paper. An equivalent version of the MLSE receiver, using a matched filter at the receiver input, was developed by Gottfried Ungerboeck of IBM Research [20]. The first known commercial application of MLSE was in a Codex partial response single sideband 9.6 kb/s voiceband modem introduced in 1969.

While MLSE receivers provide close to optimum performance on highly dispersive channels, they have a complexity drawback. If the overall channel impulse response spans N data symbols, the number of Viterbi algorithm arithmetic operations per symbol increases exponentially with N. Where N is moderate, such as in Global System for Mobile Communications (GSM)[6] and other time-division multiple access (TDMA) second-generation digital cellular systems, MLSE is practical. However for systems with long channel impulse responses, pragmatic approaches to reducing the Viterbi algorithm's complexity include drastically limiting the number of channel states it could consider at any time, and replacing the WMF with an adaptive filter that combines with the channel response to produce a very short overall impulse response. The latter approach is used in magnetic recording systems.

The complexity and error propagation issues of MLSE and DFE, respectively, motivated searches for other nonlinear equalization techniques that would be effective for mitigating ISI on highly dispersive channels. Alan Gersho and Tong Lim at Bell Labs considered a scheme in which preliminary decisions from an equalizer are used to cancel all ISI at the output of a matched filter [21]. In recent years this idea has been extended to systems using symbol interleaving and error correcting codes, by taking into account the estimated reliability of each symbol decision emerging from a decoder and operating in an iterative fashion. At each iteration, the decoder and the equalizer/canceller exchange information about the "likelihoods" (essentially the reliability) of their respective outputs. This results in what is today called turbo equalization.

EQUALIZATION OF NONLINEAR CHANNELS

Voiceband telephone channels, especially in the analog transmission era up to the 1980s, had nonlinearities due to things like misadjusted companders, which, in combination with channel bank filters, could cause a certain amount of nonlinear ISI. This was a limiting impairment for data rates at and above 9600 b/s, and could not be countered by linear equalizers, DFEs, or MLSE. Nonlinear ISI could be modeled as a Volterra expansion, involving powers and cross-products of adjacent data symbols, as well as linear terms. A 1971 patent by T. Arbuckle proposed an equalizer with such nonlinear operations operating on baseband sampled received waveforms. A *BSTJ* paper by me in September 1978 extended this approach to adaptive equalization mitigation of nonlinear ISI in quadrature modulated systems with DFE. The equalization and adaptation complexity was formidable. Fortunately, the nonlinearity problem for voiceband telephone channels tended to disappear with the replacement of old channel banks in the network. Recently, some of the nonlinearity cancellation approaches have been proposed for the problem of power amplifier nonlinearities in digital wireless systems.

FAST ADAPTATION ALGORITHMS

A requirement that arose for some always connected modems, operating in a polling mode on private voiceband lines, is fast startup — the ability of the equalizer to adapt to the channel within a few symbol intervals, in order to minimize overhead for transmission of short messages or packets. A related requirement exists for equalizers in some digital radio systems — to adapt quickly to rapid changes in the channel response. The LMS and earlier adaptation algorithms typically require a number of symbol intervals to converge that is on the order of 10 or more times the number of equalizer taps [22, 23]. Their convergence time is also prolonged for highly dispersive channels. A solution to this problem was offered by Dominique Godard, in the form of an equalizer adaptation algorithm based on Kalman filtering [24]. It was soon realized that this is equivalent to a recursive least squares (RLS) algorithm, which iteratively minimizes the time-average of the square of the error between the equalizer output and the desired output (e.g., a known training symbol or a receiver decision). The adapting tap coefficients at each iteration *exactly minimize* the sum of squared errors up to that iteration. This is in contrast to the LMS algorithm, which only aims to minimize the squared error averaged over many data symbols.

The RLS algorithm typically requires a number of training symbols equal to about twice the number of equalizer taps, to achieve convergence, independent of the nature of the channel frequency response. A downside is its complexity: the RLS algorithm is based on matrix computations and the number of arithmetic operations per iteration is proportional to the square of the number of taps (the LMS algorithm's complexity is linear in the number of

[4] *The WMF was implementable as the forward filter of an adaptive decision feedback equalizer, and the overall response used by the Viterbi algorithm was the response of the feedback filter of the DFE [19].*

[5] *Partial response is a form of fixed transmitter filtering that produces a compact transmitted spectrum at the expense of additional ISI that must be equalized.*

[6] *With a GSM symbol rate of 270 kHz and radio channel impulse response durations typically less than 10 to 20 μs,* N *is on the order of 2 to 5.*

taps). For large numbers of taps, this significantly affects the modem complexity and power consumption.

In 1976 Lennart Ljung became Professor of Automatic Control at Linköping University in Sweden, arriving from Stanford University, where he had worked with Tom Kailath and Martin Morf on low-complexity recursive algorithms for mean square regression and estimation problems. At the same time I spent a year as visiting professor at Linköping, and worked with him and Martin Morf to apply that work to develop a so-called fast Kalman or fast RLS algorithm for the adaptation of decision feedback equalizers [25, 26]. Its complexity, while greater than that of the LMS algorithm, increases only linearly with the number of equalizer tap coefficients. It is simply the RLS algorithm done with more efficient arithmetic processing, by exploiting certain properties of the matrices used in the regular RLS algorithm. There followed a flurry of research results in the 1980s on fast adaptation algorithms for transversal and lattice-structured equalizers and other adaptive systems. One of the main problems addressed in that later work by John Cioffi, Thomas Kailath, Dirk Slock at Stanford University, and others, was numerical stability of fast RLS algorithms, related to their digital implementation with finite word lengths. Fuyun Ling and John Proakis of Northeastern University published corresponding work for lattice-structured equalizers in the 1980s. Numerical precision problems persist if RLS and related fast-adapting algorithms run without stopping, on uninterrupted streams of data. Fortunately, most data traffic is in packets or frames, giving opportunities to re-start and refresh adaptation algorithms frequently.

Another practical approach to fast equalizer initial convergence was *cyclic equalization*, developed independently in a 1973 patent by G. David Forney, Jr. at Codex and by Kurt Mueller and David Spaulding at Bell Labs in a June 1974 paper at the International Conference on Communications. The idea is to transmit a *periodic* training sequence, whose period is equal to the number of equalizer taps. There are several benefits:

- The periodicity gives somewhat faster convergence than for non-periodic training sequences.
- Sequence synchronization is trivial: simply cyclically shift the converged equalizer taps so that the largest tap coefficient is in the middle.
- Faster-than-real-time adaptation is possible, using one period of the stored received sequence.
- The periodicity property facilitates equalizer tap computations using fast Fourier transform techniques.

On the other hand, the converged tap coefficients from periodic training have to be further adapted slightly once random data starts. Also, the necessity that the number of equalizer taps equals the training sequence period may introduce undesirable constraints on transmitting and receiving modems; for example, when the modems are from different manufacturers.

BLIND EQUALIZER ADAPTATION

Bob Lucky's 1966 adaptive equalizer algorithm was an example of "blind" adaptation, since it used actual receiver decisions rather than an initial sequence of known training symbols. Yoichi Sato of NEC Central Research Laboratories generalized Lucky's blind algorithm to QAM and other modulated systems, calling it a "self-recovering" adaptation algorithm [27]. Blind equalizer adaptation, without requiring explicit training symbols, confers the advantage of lower transmission overhead. It is also valuable in applications to multipoint networks and to other situations where the insertion of training symbols after disrupted transmission is not possible. A pioneering paper by Dominique Godard of IBM introduced the constant modulus (CM) algorithm [28], which attempts to minimize the MSE between a power of the *magnitude* of the equalizer's output and a constant. The CM algorithm is a relatively simple modification of the LMS algorithm, does not make use of training symbols or receiver decisions, and applies to a variety of modulation types, including multilevel ones. The downside of the CM and other blind adaptation algorithms is that they converge much slower than algorithms that start by using known training symbols. Much research continues on blind adaptation algorithms to remedy this problem. A recent lookup of "blind adaptation" on IEEE Xplore yielded over 2000 hits.

INTERFERENCE SUPPRESSION

Equation 2, representing ISI as a linear combination of interfering data symbols at a particular time instant kT, could also be a representation of interference from a different (co-channel interfering) transmitter with the same data symbol rate if the summation does not exclude the index k. It should not be surprising, then, that equalization techniques can be used to suppress not only ISI but also synchronous co-channel interference (CCI) in some circumstances. This was recognized by D. A. Shnidman in a November 1967 *BSTJ* paper that gave a generalized version of the Nyquist criterion for the combination of ISI and CCI, and also generalized Tufts' 1965 paper on equalizer optimization to the joint ISI/CCI problem. Later *IEEE Transactions on Communications* papers by Kaye and George at Carleton University in 1970 and Van Etten at Eindhoven University in 1975 applied this concept to LMS-adaptive ISI and CCI suppression in multichannel and diversity digital transmission systems. Adaptive synchronous CCI suppression has been applied to far-end crosstalk suppression in multipair cables, and interference suppression in code-division multiple access (CDMA) and non-CDMA cellular radio systems.

Adaptive linear CCI suppression can be augmented by adaptive *cancellation* of interference from known or estimated data symbols, analogous to DFE. Applications are to echo cancellation, near-end crosstalk suppression in multipair cables, and iterative CCI cancellation in radio systems. Interference cancellation and suppression can be advantageously combined with spatial processing in multi-antenna systems.

FREQUENCY DOMAIN EQUALIZATION

Time domain equalization — the convolution processing of received signal samples and symbol decisions by transversal filters — was the primary equalization paradigm from the 1960s to the 1990s. Toward the end of that period, with continuing data rate increases over wired, radio, and other transmission media, channel impulse responses spanned more and more data symbols. The number of equalizer tap coefficient multiplications involved in each data symbol decision grew accordingly. Processing *blocks* of incoming signals in the *frequency domain* provides a way of softening this tyranny of computational growth with data rate. The essential step that made this possible was provided by Steve Weinstein, Paul Ebert, and Jack Salz at Bell Labs in 1969, who realized that the then recently developed fast Fourier transform (FFT) could significantly reduce the complexity involved in frequency domain filtering of blocks of signal samples [29, 30]. Converting to the frequency domain by an FFT, frequency domain filtering, and then converting back to the time domain with an inverse FFT requires a computational effort roughly proportional to the logarithm of the channel response length. Compared to the linear growth in computational effort for time domain equalization, this yields a significant savings in signal processing complexity and power consumption when channel responses span hundreds of data symbol intervals.

Terry Walzman of Bell Labs and Mischa Schwartz of Brooklyn Polytechnic Institute applied FFT equalization to received blocks of serially transmitted data symbols interspersed with sequences of zeroes so as to preserve the cyclic property of FFTs [31]. They also described a version of LMS adaptation for the resulting frequency domain equalizer. Earl Ferrara of Stanford University in 1980 and Gregory Clark of Lawrence Livermore Laboratory, Sanjit Mitra of University of California Santa Barbara and Sydney Parker of the Naval Postgraduate School in 1981 showed how frequency domain equalization could be carried out and adapted, without requiring the interspersing of zeroes or cyclic prefixes between blocks, by use of "overlap-save" or overlap-add" processing. The processing is still on blocks of successive received samples, but successive blocks overlap.

Interest in and applications of frequency domain equalization for serial or "single carrier" transmission has been re-awakened in the last decade for fourth-generation wireless systems. FFT-based frequency domain processing is also the basis for the current popularity of orthogonal frequency-division multiplexing (OFDM) for broadband wireless systems and discrete multitone (DMT) systems for digital subscriber loop systems. The history of OFDM and its many contributors has been recounted in the comprehensive November 2009 history article in this magazine by Steve Weinstein.

EQUALIZATION OF OTHER TYPES OF CHANNELS

EQUALIZING RADIO AND UNDERWATER CHANNELS

In contrast to wire or fiber transmission channels, whose time variations, if any, are caused by things like temperature changes, radio or underwater acoustic channels may undergo relatively rapid time variations due to movement of transmitters or receivers in a field of reflectors or scatterers, which may themselves be moving. Nevertheless, many of the equalization techniques pioneered for voiceband telephone channels also helped in subsequent rapid developments in equalization of radio and underwater channels.

Equalization techniques proved necessary for military HF and tropospheric scatter radio systems and for second and later generation digital cellular radio systems. Interest in equalization for underwater acoustic telemetry systems started in the 1980s, as described in a January 1991 overview paper in the *IEEE Journal of Oceanic Engineering* by John Proakis. For HF radio channels the reflectors and scatterers are ionospheric layer boundaries; for cellular radio channels at higher frequency bands, they are solid objects in the environment; the reflectors and scatterers in underwater systems are temperature and salinity boundaries. The presence of several strong reflected received signal components can create deep nulls in channel frequency responses. For such channels, nonlinear equalization such as DFE or MLSE, is generally a must, as is the ability to adapt rapidly to channel time variations.

The effectiveness of DFE for fading dispersive channels was demonstrated by Peter Monsen [32]. Frank Hsu of GTE Sylvania, with colleagues A. Giordano, H. De Pedro and John Proakis of Northeastern University, applied square root versions of RLS adaptation algorithms to equalization of rapidly fading HF channels [33]. In the ensuing decades, nonlinear and fast-adapting RLS and other equalization techniques have been applied to cellular radio and other wireless channels.

The ability of equalizer adaptation algorithms to track and compensate for time-varying channel impulse responses depends on how rapid is the time variation relative to the data symbol rate, and also on how many equalizer tap coefficients are being adapted. The larger the number of tap coefficients the slower the equalizer can adapt and track. A rough empirical measure of adaptation ability is

$$\frac{\begin{pmatrix}\text{bandwidth of}\\ \text{time variation}\end{pmatrix}\cdot\begin{pmatrix}\text{number of}\\ \text{tap coefficients}\end{pmatrix}}{(\text{symbol rate})}.$$

If this ratio is less than about 0.01, the variation can be considered "slow," and a relatively simple LMS algorithm is probably adequate, with a careful choice of the step size parameter. Higher values of this ratio raise concerns about possible performance degradations. For HF systems, typical fading bandwidths are up to 1 to 10 Hz, symbol rates about 2 kHz, and the multipath structure calls for something on the order of 10 to 20 taps; thus, the worst case channel variation can be described as "fast." For the GSM cellular system, with a 270 kHz symbol rate and Doppler frequencies[7] of up to 200 Hz, the channel variation can be described as "borderline fast." Interestingly, these days, as multi-megabit per second wireless systems are being developed, the channel time variation is "slow" relative to the data rate. The time variability issue is much less critical than for HF radio systems whose data rates are a few kilobits per second. Advanced wireless systems use block frequency domain transmission and reception techniques. Channel estimation and adaptation can take place on a block-by-block rather than symbol-by-symbol timescale.

EQUALIZATION OF DIGITAL SUBSCRIBER LOOP CHANNELS

In 1980, at the end of the history window spanned by this article, access systems using digital transmission on twisted-pair subscriber loops were just getting started, with typical data rates of 64 to 144 kb/s, which have since increased to the 10 to 100 Mb/s rates of today. Characteristics of these channels calling for equalization include attenuation that increases with frequency and distance, bridged taps, wire gauge transitions, and transformer coupling that blocks DC. Impulse responses typically have very long duration tails, with bumps caused by the bridged taps. DFE has typically been used, with forward filters having relatively few taps, and with feedback filters that were very long. Later, asynchronous digital subscriber line (ADSL) systems for consumer subscriber line access used discrete multitone transmission (DMT) — a version of OFDM. John Cioffi provided a good review of progress on equalization for digital subscriber loops in a May 2011 history article in this magazine.

EQUALIZATION OF OPTICAL FIBER CHANNELS

As summarized in an overview paper by K. Azadet *et al.* in the March 2002 *IEEE Journal on Solid State Circuits*, optical fiber transmission line impairments include chromatic dispersion, differential and polarization mode dispersions, and birefringence effects, which can require equalization on long lines. Due to data rates of tens of gigabits per second, equalizers for optical fibers are today typically implemented with analog delay lines and multipliers, controlled by digital adaptation circuits. MLSE equalization can be applied to noncoherent detection of intensity-modulated fiber optic signals.

MAGNETIC RECORDING EQUALIZERS

Digital magnetic recording and reading heads, used in computer hard disks, cause a differential filtering process on bits during the reading process. In the late 1960s, Hisashi Kobayashi of IBM pointed out that this distortion is equivalent to linear distortions which cause ISI in digital communications systems. He also suggested that feasible equalization techniques, combined with the write/read distortions, yielded the equivalent of a partial response channel. As recounted in his 2009 paper surveying the history of equalization and coding for magnetic recording, Kobayashi took a leave of absence to work for a year at UCLA with Andy Viterbi, and learned about the Viterbi algorithm for convolutional codes. He applied it to the detection process for the partial response magnetic recording channel, producing a special case of MLSE, apparently independent of Dave Forney, who was doing the same thing at Codex, which eventually led to Forney's classic MLSE paper in 1972. Since then, partial response maximum likelihood (PRML) coding and detection methods have been a mainstay in magnetic recording technology.

POST-1980 IMPACTS OF ADAPTIVE EQUALIZATION ALGORITHMS AND TECHNOLOGY

Equalizer research resulting in new algorithms, refinements of old algorithms, implementation innovations, and analysis of equalization performance continues apace. Advances in equalization algorithms and theory have played a very large role in the data rate increases achieved in wireline and wireless modems in recent decades. A search of IEEE Xplore reveals over 1500 papers involving equalization since the beginning of 2009. The family of blind equalization algorithms continues to flourish. The advent of new generations of wireless access systems has spurred advances in equalization coupled with spatial processing, for diversity and interference rejection, as well as for ISI suppression. This also has application in the suppression of interference in multipair cable systems. Turbo equalization is realized as a practical solution to the problem of efficient reception of coded signals from frequency selective channels. Within all these research threads, frequency domain equalization continues to proliferate, and exploits synergy with corresponding research on OFDM techniques.

ACKNOWLEDGMENTS

I am very grateful for helpful comments received from John Cioffi, Dave Forney, Jerry Hayes, Bob Lucky, John Proakis, and Gottfried Ungerboeck.

REFERENCES

[1] J. G. Proakis and J. H. Miller, "An Adaptive Receiver for Digital Signaling Through Channels with Intersymbol Interference," *IEEE Trans. Info. Theory*, vol. IT-15, Apr. 1969, pp. 484–97.
[2] S. U. H. Qureshi, "Adaptive Equalization," *Proc. IEEE*, vol. 73, Sept. 1985, pp. 1349–87.
[3] R. W. Lucky, J. Salz, and E. J. Weldon, Jr., *Principles of Digital Communication*, McGraw-Hill, 1968.
[4] J. G. Proakis, *Digital Communications*, 1st and later eds., McGraw-Hill, 1983.
[5] R. D. Gitlin, J. F. Hayes, and S. B. Weinstein, *Data Communications Principles*, Plenum Press, 1992.
[6] H. Nyquist, "Certain Factors Affecting Telegraph Speed," *Bell Sys. Tech. J.*, vol., 3, Apr. 1924, pp. 324–46.
[7] H. Nyquist, Certain Topics in Telegraph Transmission Theory," *Trans. Am. Inst. of Elec. Eng.*, vol. 47, Apr. 1928, pp. 617–44.
[8] D. W. Tufts, "Nyquist's Problem — The Joint Optimization of Transmitter and Receiver in Pulse Amplitude Modulation," *Proc. IEEE*, Vol. 53, No. 3, Mar. 1965, pp. 248–59.
[9] R. W. Lucky, "Automatic Equalization for Digital Communication," *Bell Sys. Tech. J.,* vol. 44, Apr. 1965, pp. 547–88.
[10] R. W. Lucky, "Techniques for Adaptive Equalization of Digital Communication," *Bell Sys. Tech. J.,* vol. 45, Feb. 1966, pp. 255–86.
[11] B. Widrow and M. E. Hoff, "Adaptive Switching Circuits," *Proc. IRE WESCON Convention Record*, Part 4, Aug. 1960, pp. 96–104.
[12] D. C. Coll and D. A. George, "A Receiver for Time-Dispersed Pulses," *Conf. Record, 1965 IEEE Int'l. Conf. Commun.*, pp. 753–58.
[13] R. D. Gitlin, E. Y. Ho, and J. E. Mazo, "Passband Equalization of Differentially Phase-Modulated Data Signals," *Bell Sys. Tech. J.*, vol. 52, Feb. 1973, pp. 219–38.
[14] H. Kobayashi, "Simultaneous Adaptive Estimation and Decision Algorithm for Carrier Modu-

[7] *The Doppler frequency gives a rough measure of the bandwidth of the channel time variation. It is proportional to the relative velocity, normalized by the speed of light, and to carrier frequency.*

lated Data Transmission Systems," *IEEE Trans. Commun. Tech.*, vol. COM-19, June 1971, pp. 268–80.

[15] R. Price, "Nonlinearly Feedback-Equalized PAM Vs. Capacity for Noisy Filter Channels," *Proc. ICC 1972,* May 1972, pp. 22-12–22-17.

[16] D. A. George, R. R. Bowen, and J. R. Storey, "An Adaptive Decision Feedback Equalizer," *IEEE Trans. Commun. Tech.*, vol. COM-19, June 1971, pp. 281–93.

[17] M. Tomlinson, "New Automatic Equalizers Employing Modulo Arithmetic," *Elect. Lett.*, vol. 7, Mar. 1971, pp. 138–39.

[18] H. Harashima and H. Miyakawa, "Matched Transmission Technique for Channels with Intersymbol Interference," IEEE Trans. on Commun. Tech., vol. COM-20, Aug. 1972, pp. 774–80.

[19] G.D. Forney, Jr., "Maximum-Likelihood Sequence Estimation of Digital Sequences in the Presence of Intersymbol Interference," *IEEE Trans. Info. Theory*, vol. IT-18, May 1972, pp. 363–78.

[20] G. Ungerboeck, Adaptive Maximum-Likelihood receiver for Carrier-Modulated Data Transmission Systems," *IEEE Trans. Commun.*, vol. COM-22, May 1974, pp. 624–36.

[21] A. Gersho and T.L. Lim, "Adaptive Cancellation of Intersymbol Interference for Data Transmission," *Bell Sys. Tech. J.*, vol. 60, Nov. 1981, pp. 1997–2021.

[22] G. Ungerboeck, "Theory on the Speed of Convergence in Adaptive Equalizers for Digital Communication," *IBM J. Res. and Dev.*, vol. 16, Nov. 1972, pp. 546–55.

[23] J.E. Mazo, "On the Independence Theory of Equalizer Convergence," *Bell Sys. Tech. J.*, vol. 58, May-June 1979, pp. 963–93.

[24] D.N. Godard, "Channel Equalization Using a Kalman Filter for Fast Data Transmission," *IBM J. Res. and Dev.*, Vol. 18, May 1974, pp. 267–73.

[25] L. Ljung, M. Morf, and D. Falconer, "Fast Calculation of Gain Matrices for Recursive Estimation Schemes," *Int'l. J. Control*, vol. 27, Jan. 1978, pp. 1–19.

[26] D. D. Falconer and L. Ljung, "Application of Fast Kalman Estimation to Adaptive Equalization," *IEEE Trans. Commun.*, vol. COM-26, Oct. 1978, pp. 1439–46.

[27] Y. Sato, "A Method of Self-Recovering Equalization for Multi-Level Amplitude Modulation Systems," *IEEE Trans. Commun.*, vol. COM-23, June 1975, pp. 679–82.

[28] D. N. Godard, "Self-Recovering Equalization and Carrier Recovery in Two-Dimensional Data Communication Systems," *IEEE Trans. Commun.*, vol. COM-28, Nov. 1980, pp. 1867–75.

[29] J. Salz and S. B. Weinstein, "Fourier Transform Communication System," *Proc. ACM Conf. Comp. and Commun.,* Pine Mountain, GA, Oct. 1969.

[30] S. B. Weinstein and P. M. Ebert, "Data Transmission for Frequency-Division Multiplexing Using the Discrete Fourier Transform," *IEEE Trans. Commun. Tech*, Vol. COM-19, Oct. 1971, pp. 628–34.

[31] T. Walzman and M. Schwartz, "Automatic Equalization Using the Discrete Frequency Domain," *IEEE Trans. Info. Theory*, vol. 19, Jan. 1973, pp. 59–68.

[32] P. Monsen, "Feedback Equalization for Fading Dispersive Channels," *IEEE Trans. Info. Theory*, vol. IT-17, Jan., 1971, pp. 56–64.

[33] F. M. Hsu et al., "Adaptive Equalization Techniques for High Speed Data Received over Fading Dispersive HF Channels," *Proc. Nat'l. Telecom. Conf. 1980,* Houston, TX, 1980.

BIOGRAPHY

DAVID FALCONER [LF] received his B.A.Sc. degree in engineering physics from the University of Toronto in 1962, S.M. and Ph.D. degrees in electrical engineering from M.I.T. in 1963 and 1967, respectively, and an honorary doctorate of science from the University of Edinburgh in 2009. After a year as a postdoctoral fellow at the Royal Institute of Technology, Stockholm, Sweden he was with Bell Laboratories from 1967 to 1980 as a member of technical staff and group supervisor. During 1976–1977 he was a visiting professor at Linköping University, Linköping, Sweden. Since 1980 he has been with Carleton University, Ottawa, Canada, where he is now Professor Emeritus and Distinguished Research Professor in the Department of Systems and Computer Engineering. His recent research interests have centered around beyond-third-generation broadband wireless communications systems. He was director of Carleton's Broadband Communications and Wireless Systems (BCWS) Centre from 2000 to 2004. He was the Chair of Working Group 4 (New Radio Interfaces, Relay-Based Systems and Smart Antennas) of the Wireless World Research Forum (WWRF) in 2004 and 2005. He received the 2008 Canadian Award for Telecommunications Research, a 2008 IEEE Technical Committee for Wireless Communications Recognition Award, the IEEE Canada 2009 Fessenden Award (Telecommunications), and the IEEE Communications Society Award for Public Service in the Field of Telecommunications.